

MATH 3 - FINAL EXAM – K. HOGUE
Fall 2023

200 POINTS

NAME: _____

SHOW ALL WORK NEATLY AND CLEARLY MARK YOUR ANSWERS.

Fill in the blanks with the most appropriate answer.

(3 points each)

(1) Simplify: $\frac{13x^4y^3}{26x^{-3}y^8}$ _____

(2) Simplify $(3 + \sqrt{x})(3 - \sqrt{x})$ _____

(3) Simplify completely: $(-3x^{1/3}y^{-1})(11x^{1/2}y^4)$ _____

(4) If $f(x) = |x| + x$, what does $f(x)$ simplify to when $x < 0$? _____

(5) If $f(x) = \sqrt{x}$, find $f(x+h)$ _____

(6) Simplify. $\frac{2x}{x+5} + \frac{6}{x^2 - 2x - 35}$ _____

(7) Factor. $10x^2 + 17x + 3$ _____

(8) The slope of a line parallel to the line $5x+4y=7$ is _____

(9) Factor $x^3 - 8$ _____

(10) Simplify $\frac{2x^2}{3x^2 + 5xy}$ _____

CIRCLE T FOR TRUE, F FOR FALSE. (2 points each)

T F (11) If $\sqrt{64} = \pm 8$

T F (12) $\left(\frac{5x}{z^3}\right)^{-3} = \frac{z^9}{125x^3}$

T F (13) $(x-y)^2 = (x-y)(x+y)$

T F (14) $\sqrt{x^4+9} = x^2+3$

T F (15) $\sqrt{x^2} = x$ for all x

(16) Find the equation of the line containing the points (5,1) and (-3,2)

(10 points)

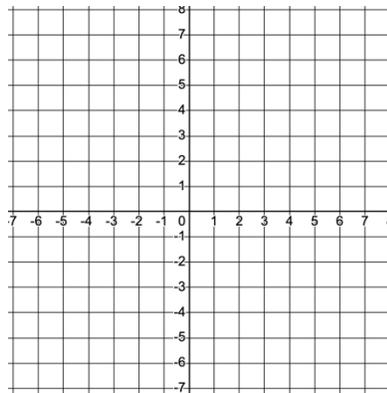
(17) Find the domain for each of the following functions. Express answer as interval.

(5 points each)

(a) $f(x) = \sqrt{x+1}$

(b) $g(x) = \frac{x}{x^2+7x+10}$

(18) Graph $\begin{cases} 2x+1 & \text{if } x \leq 0 \\ |x-3| & \text{if } x > 0 \end{cases}$ Label 2 points on graph. (10 points)



(19) Given $f(x) = x^2$, find $f(2a) - 2f(a)$

(6 points)

(20) Factor Completely: (6 points each)

(a) $12x^{1/3} - 3x^{7/3}$

(b) $3(x^2 + 4)^2 2x(x+5)^6 + (x^2 + 4)^3 6(x+5)^5$

(21) Simplify: (6 points each)

(a)
$$\frac{2\sqrt{3+x} - \frac{2x}{\sqrt{3+x}}}{3+x}$$

(b)
$$\frac{a^{-2} - b^{-2}}{b - a}$$

(22). Find all the solutions of the following and simplify:

(9 points each)

(a) $3x^2 + x - 1 = 0$

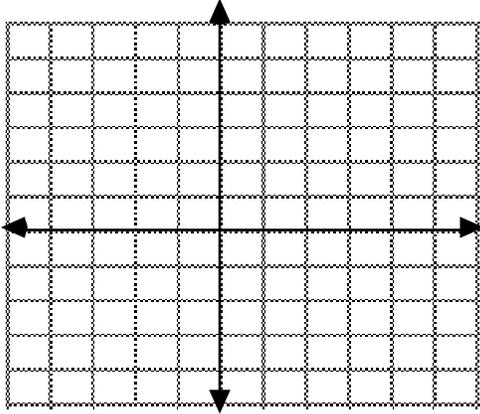
(b) $x^2 - x - 30 \leq 0$

(c) $\sqrt{x} - \sqrt{x-3} = 1$

(d) $x^4 - 7x + 12 = 0$

(23) Given the function $f(x) = \frac{1}{2}x^2 + 4x + 6$ (8 points)

put $f(x)$ in the form $f(x) = a(x-h)^2 + k$ and sketch the graph. On the graph label the vertex plus one other point.



(24) Given the graph of $y = f(x)$ (16 points)

(a) Express answers using interval notation:
-make it clear if you are using (vs. [

Domain of $f(x)$? _____

Range of $f(x)$? _____

Where is $f(x)$ decreasing ? _____

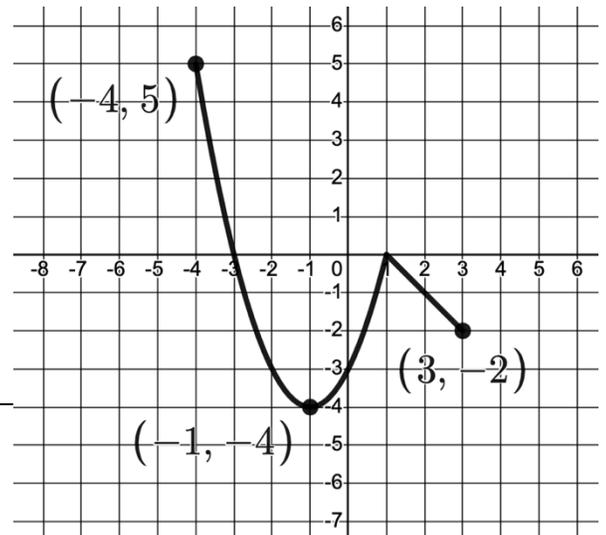
(b) Find the coordinates of local max(s), if any _____
(recall, local extrema do not occur at endpoints)

(c) What is the value of $f(2)$? _____

(d) Find a value of a for which $f(a) = 5$ _____

(e) What are the y intercept(s) (if any) _____

(f) On the axes given, graph $y = -f(x - 1)$



(25) Find the inverse of $f(x) = \frac{5x}{x-1}$. What is the domain and the range of $f(x)$? (10 points)

(26) Given the polynomial , $f(x) = x^3 - 3x^2 + 4$ (12 points)

(a) find end behavior _____

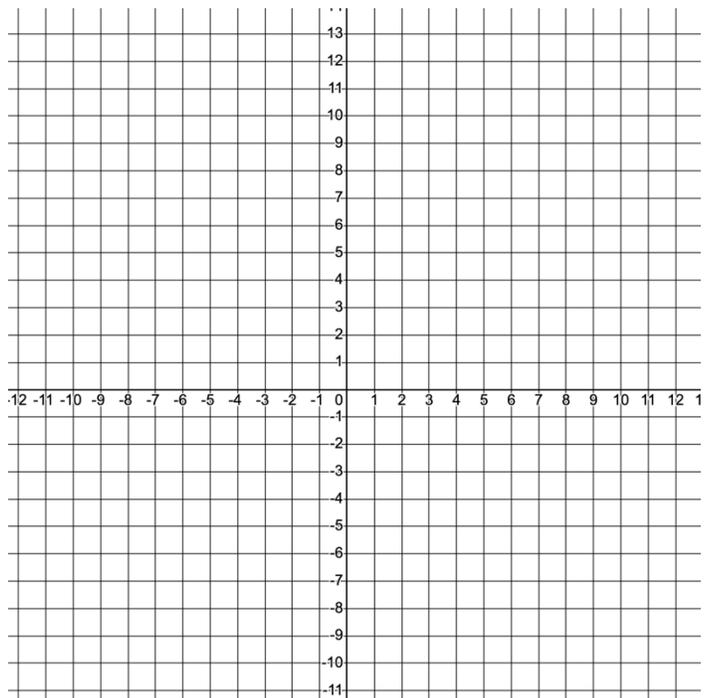
(b) find the y intercept _____

(c) find the list of possible rational zeros of f(x) _____

(d) find the x intercepts and discuss the behavior near them.

(e) plot one additional point for accuracy and sketch the graph. Label two points

SHOW ALL WORK



(27) Match the function to its graph:

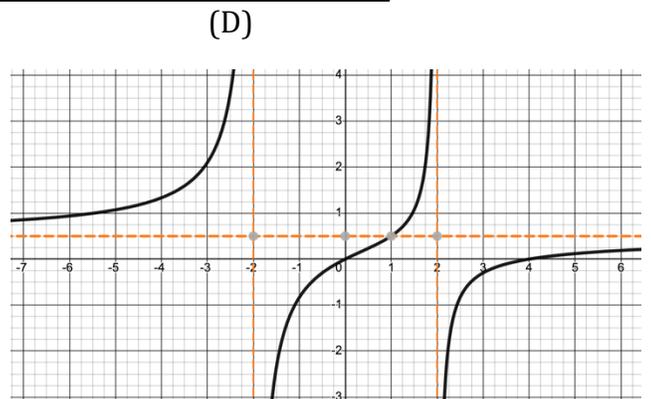
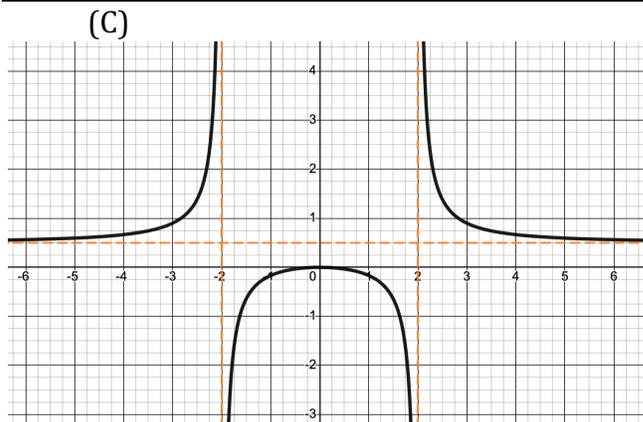
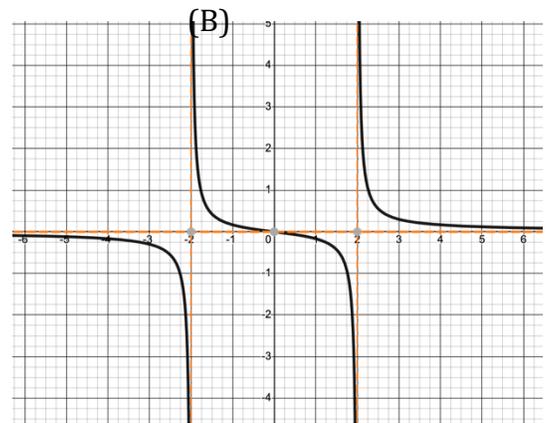
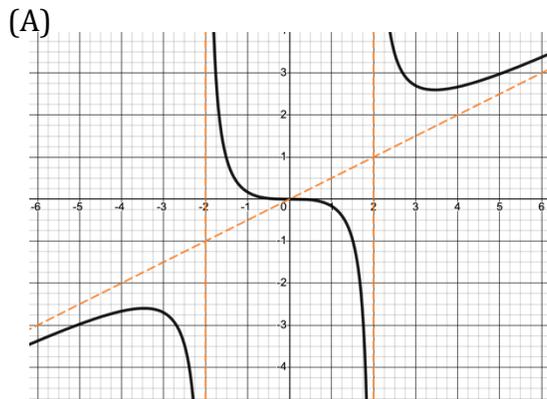
(8 points)

a) $f(x) = \frac{x}{2x^2 - 8}$ _____

b) $f(x) = \frac{x^3}{2x^2 - 8}$ _____

c) $f(x) = \frac{x^2}{2x^2 - 8}$ _____

d) $f(x) = \frac{x(x-4)}{2x^2 - 8}$ _____



(28) A long, rectangular sheet of metal 12 inches wide is to be made into a rain gutter by turning up two sides so that they are perpendicular to the sheet. How many inches should be turned up to give the gutter the greatest capacity. (Let x be the length of the sides turned up)

(10 points)

